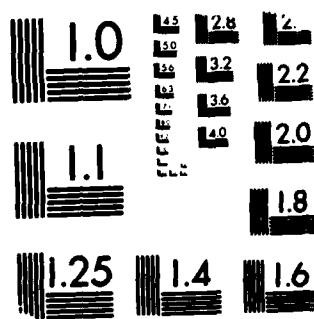


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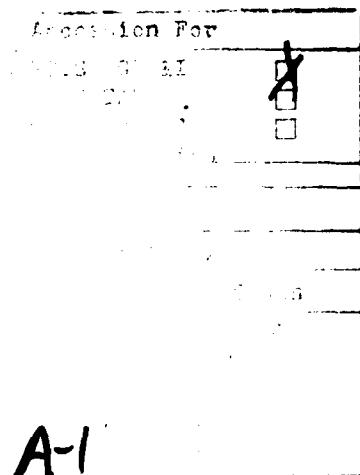
MEMORANDUM FOR DIRECTOR, SURFACE WARFARE DIVISION (OP-32)
Attn: Commander R. I. McManus (OP-324C)

Subj: Center for Naval Analyses Research Memorandum 87-111

Encl: (1) CNA Research Memorandum 87-111, "Modeling Failures of Intermittently Used Machines," by Dean A. Follmann, July 1987

1. Enclosure (1) is forwarded as a matter of possible interest.
2. This Research Memorandum presents methodology for analyzing failures of machines that are repeatedly turned on and off. Because a machine can fail both when it is on and off, different parametric models for failure are used for each of these periods. An important issue addressed for such machines is how the intermittent use itself affects failure. Because the models can predict the change of failure under different usage patterns, less harmful usage patterns can be recommended. As an example, the models are applied to a radar system, and both the immediate and cumulative effects of on-off cycling are demonstrated.

Peter J. Evanovich
Director
Readiness and Sustainability Program



A-1

CRM 87-111 / July 1987

MODELING FAILURES OF INTERMITTENTLY USED MACHINES

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ABSTRACT

This research memorandum presents methodology for analyzing failures of machines that are repeatedly turned on and off. Because a machine can fail both when it is on and off, different parametric models for failure are used for each of these periods. An important issue addressed for such machines is how the intermittent use itself affects failure. Because the models can predict the chance of failure under different usage patterns, less harmful usage patterns can be recommended. As an example, the models are applied to a radar system, and both the immediate and cumulative effects of on-off cycling are demonstrated.

TABLE OF CONTENTS

	Page
Introduction	1
Model	3
Model Validation	11
Example	12
Discussion	20
References	21

LIST OF ILLUSTRATIONS

	<u>Page</u>
1 Two On-Off Patterns Ending in Failure	4
2 Three Hazard Functions for the Two-Weibull Model	8
3 Actual Usage History for One Radar	13
4 Estimated Hazard Function for a Daily-On/Daily-Off Pattern	16
5 Pearson Residual by Grouping Number	18
6 Estimated Survival Function of the Residuals	19

INTRODUCTION

Many machines are used intermittently, that is, repeatedly turned on and off during normal operation. Eventually, the machine fails. Following failure, the machine may be repaired or replaced. Failures can occur either when a machine is on or off—an off period might be followed by an unsuccessful attempt to turn the machine on. Examples include light bulbs, automobiles, and electronic components.

Although some work has been done on modeling repairable machines [1 and 2], relatively little work has been directed towards modeling intermittently used machines. There are indications, however, that on-off cycling has a harmful effect on a system's reliability. As demonstrated in [3], the failure rate increases with on-off cycling for certain intermittently used systems. Reference [2] suggests that on-off cycling may be associated more strongly with failures than with operating time. Modeling failures for such machines requires some modification of the usual methodology under which the time until failure is regarded as a continuous random variable without regard to the effect of on-off cycling.

In this analysis, the approach to modeling machine failure is to divide the time-until-failure into on and off periods and specify different parametric failure models for each period. This approach allows the risk of failure to vary over on and off periods. To estimate the parameters of these models requires a continuous history of when the machine is on, off, or broken. For on periods, a Weibull regression model on the time since repair is assumed. The discovery of failure following an off period is modeled with a logit regression. Covariates in both models are used to describe the operating history of the machine. In this way, historical usage as well as other factors are allowed to affect the chance of failure.

The models can describe both the immediate and cumulative effect on reliability of on-off cycling. Cumulative on-off cycling can be included as a covariate in both models. The immediate effect is handled differently for each. For the logit model, the intercept parameter roughly corresponds to any factor that occurs with each off period—for example, the actual shutting off and turning on. Each on period begins by switching the machine on. The risk of failure during an on period is allowed to vary as a function of the time since switching on, by incorporating a time-dependent covariate in the Weibull model. The chosen function $\exp\{\beta_0 \ln(u)\}$, where u is the time since switched on, allows risk to decrease, increase, or remain constant with u .

An important application of the models is for prediction. Using estimated models, the probability of failure under different possible usage patterns can be computed. Given that the models fit reasonably well, these probabilities can act as an aid in choosing how to use a machine under a certain set of conditions, with an eye towards improving reliability.

The next section describes the type of data necessary for estimating the Weibull and logit models. The two models are developed and model fit is discussed. Subsequently, the models are applied to a radar system, and both the immediate and cumulative effects of on-off cycling are demonstrated. The probabilities of failure under different usage patterns are computed and the most reliable usage pattern determined. Finally, generalizations and other applications of the methodology are discussed.

MODEL

Suppose a machine is repeatedly switched on and off, occasionally fails, and is repaired after failure. Figure 1 provides two possible on-off patterns that end in failure. Pattern A is for a machine that follows repair with a three-day on period and a one-day off period. At this point, an unsuccessful attempt was made to turn on the machine. Failure could have occurred anytime between days three and four. Pattern B is for a machine that is switched on or off three times and fails while working five days following repair. The history for an intermittently used repairable machine consists of numerous on-off patterns like A and B. Often, only operating time is used to describe failures, and the original patterns A and B are transformed into A' and B'. This precludes direct examination of the effect of on-off cycling on failure rates. The approach in this paper is to retain both the on and off periods and model A and B rather than A' and B'.

Clearly a machine can fail while on, but a machine can also fail while off.¹ As in pattern A of figure 1, occasionally, an off period will be followed by an unsuccessful attempt to turn the machine on. Each off period involves shutting off, turning on, and possible exposure to such factors as vibration or corrosion, which might cause failure. A reasonable model should allow failure to occur while a machine is off as well as when it is on.

The stress a machine receives while on is qualitatively different from the stress it receives while off. This difference suggests that failures should be modeled separately for each period. Apart from the different risk, the two periods also differ in the amount of failure information they provide. If a machine fails while on, the precise time to failure is known; if failure occurs while off, failure is only known to have occurred sometime while the machine was off. In other words, for on periods, the time of failure should be modeled with a continuous probability model. For off periods, the occurrence or not of failure should be modeled by a discrete 0-1 probability.

The time until failure T , therefore, is modeled as a combination of discrete and continuous components. Let $0 = t_1 < t_2 \dots$ denote the times at which a machine is switched on or off. For simplicity, assume that only on periods immediately follow repair. When j is odd, t_j marks the start of an on period (or end of an off period). When j is even, t_j marks the start of an off period (or end of an on period). As shown in [4], the survivor function for a combination of discrete and continuous components can be written as

$$P(T \geq t) = \exp \left[- \int_0^t \lambda(u) du \right] \prod_{j: t_j+1 < t} [1 - p(j)] . \quad (1)$$

¹The "off" period is used in a generic sense and includes portions of the shutting off or turning on sequence when the system is partially energized.

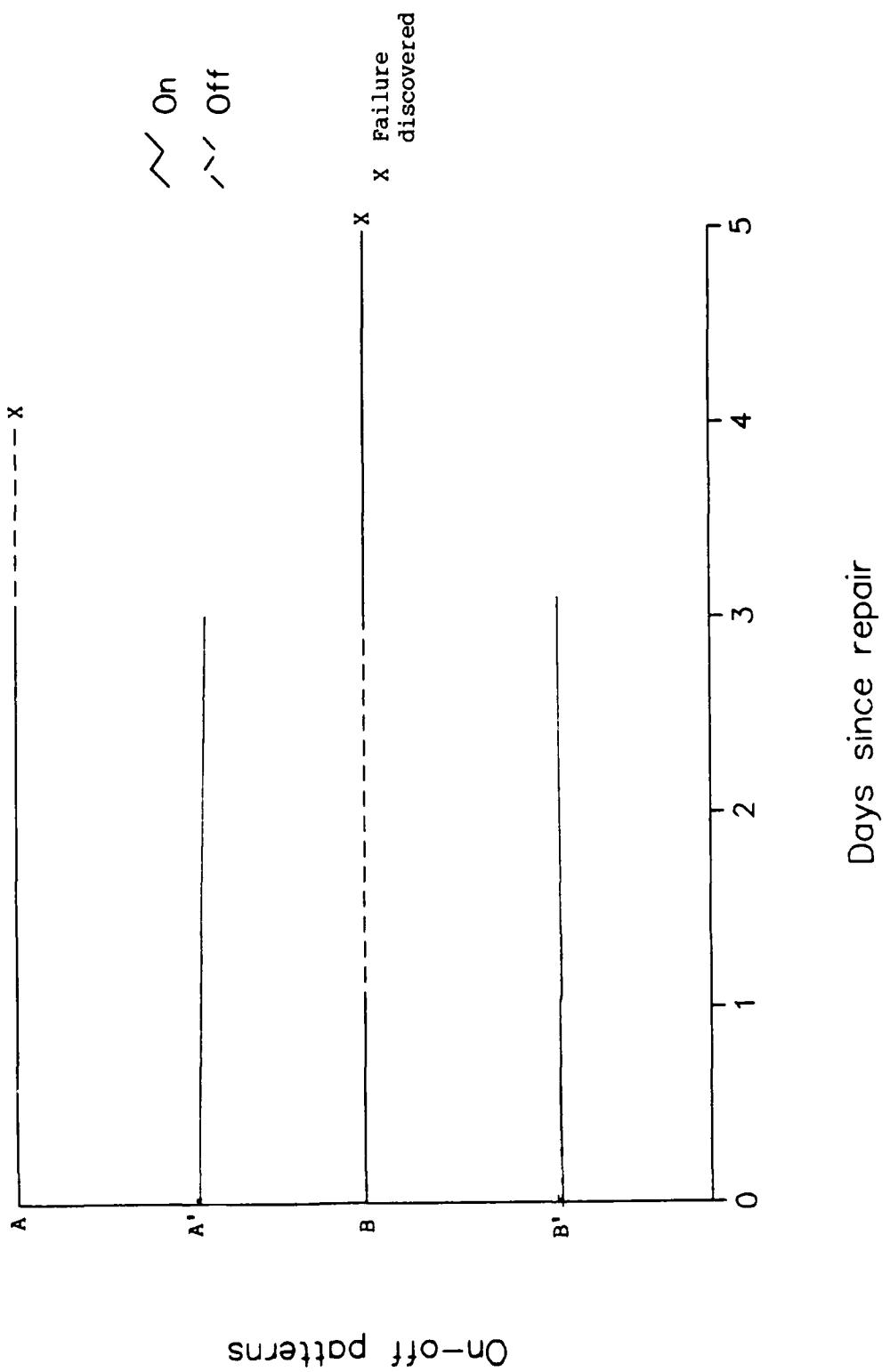


FIG. 1: TWO ON-OFF PATTERNS ENDING IN FAILURE

where

$$\begin{aligned}t_{2j+1} &= \text{the end of the } j\text{th off period} \\ \lambda(t) &= \begin{cases} \text{the hazard function while the machine is on} \\ (0 \leq t \leq t_2, \text{ or } t_{2j-1} < t \leq t_{2j} \text{ for } j > 1) \\ 0 \text{ while the machine is off} \end{cases} \\ p(j) &= \text{probability the } j\text{th off period ends in failure} .\end{aligned}$$

The overall model is specified by two separate functions. During on periods, a continuous hazard function or instantaneous rate of failure $\lambda(t)$ is used. During off periods, a discrete hazard or conditional probability of failure $p(j)$ is specified. Note that for notational convenience, off failures are ascribed to the end of an off period at t_{2j-1} . In actuality, there is no way of knowing when during the off period failure occurred. Also for notational convenience, the fact that for the first on period the hazard is defined at $t_1 = 0$ will be suppressed.

A popular choice for a continuous hazard function, which incorporates covariates, is given by the proportional hazards model [4]. Here, the hazard function is written as a product of an exponential function of covariates times a baseline hazard function. The covariates act multiplicatively on the baseline hazard function $\lambda_0(t)$:

$$\lambda\{t \mid z(t)\} = \lambda_0(t) \exp\{z(t)'\beta\} . \quad (2)$$

where

$$\begin{aligned}t &= \text{a time index} \\ \lambda_0(t) &= \text{the baseline hazard rate} \\ z(t) &= \text{a column vector of possibly time-varying covariates} \\ \beta &= \text{a column vector of covariate parameters} .\end{aligned}$$

As suggested in [1], the time index might be age of the machine or, for repairable machines, time since the last repair or operating time since last repair. For simplicity, in the remainder of the paper t will be time since last repair. The covariate vector $z(t)$ can be used to describe the usage history or other aspects of the machine. While $\lambda_0(t)$ might be left unspecified, in this paper a parametric (Weibull) baseline hazard will be used. An important application of the model is for prediction, and a well-chosen parametric hazard is somewhat easier to work with than a nonparametric hazard.

Although historical and cumulative usage may influence the chance of failure, it is also possible that switching a machine on has an "immediate" effect on the failure rate. The chance of failure may be high when the machine has just been switched on and then

taper off, or follow an opposite pattern. This suggests multiplying the baseline hazard by a function of the time since on (u), which can be either increasing or decreasing.² One such function is $\exp\{\beta_0 \ln(u)\}$. With this function of u and the Weibull baseline hazard, equation 2 can be rewritten for the j th on period as

$$\lambda\{t - z(t)\} = \theta_0 t^{\alpha_1 - 1} \exp\{\beta_0 \ln(t - t_{2j-1}) + z.(t)'\beta.\} \quad (3)$$

$$\lambda\{t - z(t)\} = \theta_0 t^{\alpha_1 - 1} u^{\alpha_2 - 1} \exp\{z.(t)'\beta.\}, \quad (4)$$

where

$$\beta' = [\beta_0, \beta_1']$$

$$\beta_0 = \alpha_2 - 1$$

$$z(t)' = [\ln(u), z.(t)']$$

$$u = (t - t_{2j-1}), \text{ for } t_{2j-1} < t \leq t_{2j}.$$

Recall that t_{2j-1} marks the start of the j th on period.

When α_1 is unity, the model reduces to a Weibull with time index u , and the start of each on period resets the time index to 0. When α_2 is unity, a Weibull with time index t results, and, in this case, there is no harmful transient effect following turning the machine on. If both α_1 and α_2 differ from unity, 4 is, in effect, the product of two Weibull models with different time scales. Alternatively, equation 4 can be viewed as a simple Weibull on either u or t with a time-dependent covariate, respectively, $\ln(t) = \ln(u - t_{2j-1})$ or $\ln(u) = \ln(t - t_{2j-1})$. Reference 4, pages 123-124, provides an example of a similar model where a Weibull shape parameter α has a "regression" interpretation, that is, as a "slope" parameter in a proportional hazards model.

Figure 2 displays the hazard rate under three special cases of this "two-Weibull" model when $z.(t) = 0$. In figure 2, the machine is assumed to be repaired, turned on for almost one day, briefly switched off, and then turned on again. Both models with $\alpha_2 = .8$ display an increase in risk following switching on at the start of the second day. This corresponds to a harmful transient effect following switching on. For the Weibull with

²Reference 5 makes a similar assumption about the transient effect of a discrete event on the hazard rate in its example 8.9. Immediately following a heart transplant, the hazard is multiplied by a decreasing function of time.

time-since-repair as time index ($\alpha_1 = .8, \alpha_2 = 1$), there is a smoothly decreasing hazard, except for the brief off period at the end of day one. For the Weibull with time-since-on as time index ($\alpha_1 = 1, \alpha_2 = .8$), the time index is set back to zero at the start of the second day, and the hazard for the second day is identical to that of the first. When both α_1 and α_2 differ from unity, there is a decreasing hazard following both switchings on; however, the decrease is less sharp following the second switching on.

Equation 4 is a model for failures while the machine is turned on. As noted before, however, failures, occasionally are discovered at the end of an off period. If a failure immediately follows an off period, the exact time of failure is unknown. The failure is known only to have occurred sometime during the off period. In other words, there is a 0-1 indicator of whether or not failure occurs associated with each off period. A logit regression model will be used to describe the probability of failure $p(x_j)$ (with a slight abuse of notation) for the j th off period:

$$\ln \frac{p(x_j)}{1 - p(x_j)} = x_j' \theta \quad , \quad (5)$$

where

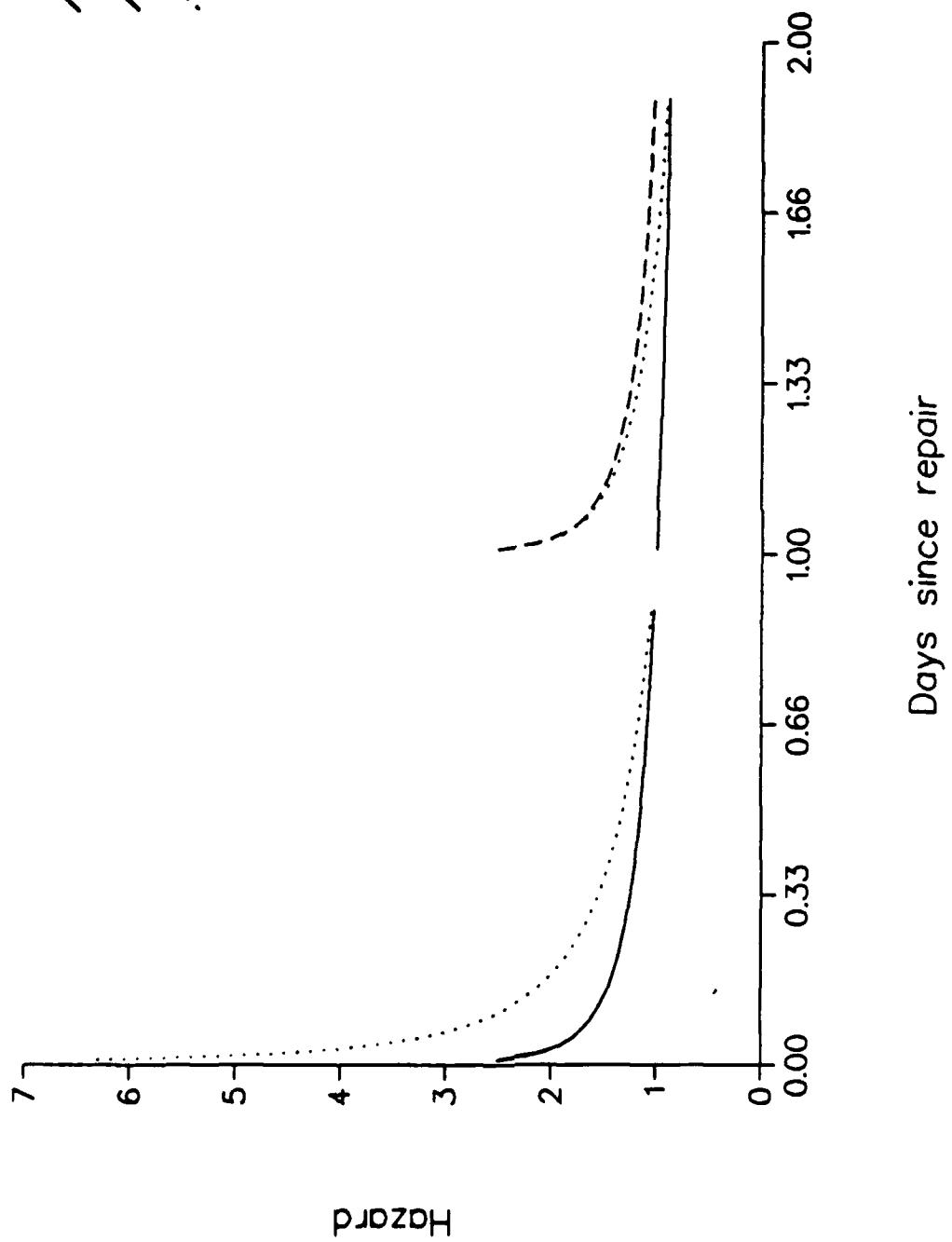
θ is a column vector of parameters

x_j is a column vector of explanatory variables that describe the j th off period

$p(x_j)$ is the probability that the j th off period ends in failure.

For both models, the covariate vectors might include fixed attributes of the machine (e.g., manufacturer) or descriptions of the on-off history of the machine, (e.g., daily rate of on-off cycling). For x_j , the length of the off period might be included to see if longer off periods are more likely to result in failure. For example, in pattern B of figure 1, one element of x_1 would be 2.

With equations 4 and 5, the likelihood for any on-off pattern can be specified. Referring again to the notional histories of figure 1, let x_1^A be the logit covariate vector associated with the off period of pattern A, and $z_A(t)$ be the covariate history for the on



period, with a similar definition for pattern B. The likelihood corresponding to the data of figure 1 is

$$\exp[-\int_0^3 \lambda\{t | z_A(t)\} dt] p(x_2^A) \times \\ \exp[-\int_0^1 \lambda\{t | z_B(t)\} dt] \{1 - p(x_1^B)\} \exp[-\int_3^5 \lambda\{t | z_B(t)\} dt] \lambda\{5 | z_B(5)\} .$$

In general, suppose there are n failures and that the on-off pattern for the i th failure has s_i off periods and r_i on periods ($s_i = r_i$ or $r_i = 1$). Let $0 = t_{i,1} < t_{i,2} \dots < t_{i,m_i}$, where $m_i = r_i + s_i - 1$, denote the start of each on or off period since the last repair, for the i th failure. Following the convention that only on periods immediately follow repair, the likelihood can be written as follows:

$$\prod_{i=1}^n \prod_{j=1}^{s_i} [1 - p(x_{i,j})]^{y_{i,j}} p(x_{i,j})^{1-y_{i,j}} \prod_{j=1}^{r_i} \exp[-\int_{A_{i,j}} \lambda\{w | z(w)\} dw] \lambda\{t_{i,2j} | z(t_{i,2j})\}^{d_{i,j}} \quad (6)$$

where

$$y_{i,j} = \begin{cases} 1 & \text{if the } i,j\text{th off period ends in failure} \\ 0 & \text{otherwise} \end{cases}$$

$$d_{i,j} = \begin{cases} 1 & \text{if the } i,j\text{th on period ends in failure} \\ 0 & \text{otherwise} \end{cases}$$

$$A_{i,j} = [t_{i,2j-1}, t_{i,2j}] \text{ the time interval for the } i,j\text{th on period} .$$

Note that for any i , only y_{i,s_i} or d_{i,r_i} (but not both) can be 1.

The likelihood can be factored into the product of the "two Weibull" parameters and the logit parameters. Maximization of the likelihood can be accomplished separately for each set of parameters. The logit likelihood is straightforward to maximize. The two-Weibull likelihood is a bit harder since it involves at least one time-dependent covariate. A brief discussion of likelihood maximization with time-dependent covariates is given in [5].

As noted before, if α_1 is unity, equation 4 reduces to a simple Weibull regression with time-since-on as the time index. With this simplification, the likelihood can be maximized using readily available software. If some covariate selection is required, it may be advantageous to use this simpler model for exploratory work. In addition, estimates based on the simpler model can be used as an initial guess for the two-Weibull maximization. For each model, the matrix of Fisher information can be inverted and the

diagonal elements used to determine the significance of the individual parameters in the usual fashion. Additionally, score or likelihood ratio tests can be used to test hypotheses about the parameters.

MODEL VALIDATION

The goodness of fit of the model can be assessed by separately examining the fit of the logit and Weibull models using generalized residuals. Generalized residuals are functions of the data that, if the model fits, roughly follow a known distribution.

Reference [6] discusses generalized residuals for logit models. When the $Y_{i,j}$'s are generally zero, they suggest grouping the observations into "cells" based on similar $x_{i,j}$ values and standardizing the number of failures in each cell. One such residual is based on the components of the Pearson chi-square goodness-of-fit statistic:

$$r_k = \frac{Y_k - E[Y_k]}{V[Y_k]^{(1/2)}} ,$$

where

$$\begin{aligned} Y_k &= \sum Y_{i,j} \\ E[Y_k] &= \sum p(x_{i,j}) \\ V[Y_k] &= \sum p(x_{i,j})[1 - p(x_{i,j})] . \end{aligned}$$

The sums are taken over the k th grouping of the $x_{i,j}$'s. If the model fits, the r_k should roughly follow a standard normal distribution.

If the Weibull model is viewed as a Weibull with time index u (time since turned on) and time-dependent covariate $\ln(u + t_{i,2j-1})$, the i, j th generalized residual can be defined as

$$\hat{F}(u_{i,j}) = 1 - \exp[- \int_0^{u_{i,j}} \hat{\lambda}\{w + t_{i,2j-1} - z(u - t_{i,2j-1})\} dw] ,$$

where $u_{i,j} = t_{i,2j} - t_{i,2j-1}$ is the length of the on period, and $t_{i,2j-1}$ is the time-since-failure at the start of the on period. According to the probability integral transformation, the $\hat{F}(u_{i,j})$'s will, if the model fits, behave like a random sample from a uniform(0,1) distribution (with some right-censored observations). A Kaplan-Meier survival-curve estimate based on the $\hat{F}(u_{i,j})$'s should look like a uniform(0,1) survival curve if the model fits [4 or 5].

EXAMPLE

One example of an intermittently used machine is a U.S. Naval radar. While at sea, the radar is generally on. While in port, the radar is generally off. Other factors that determine usage are preventive maintenance, assurance that the radar works, and ship overhaul. Although operational considerations sometimes require the radar to be on or off, at other times there may be no strong requirement for a radar to be either on or off. For example, preventive maintenance schedules might be changed or assurance checks might be done less frequently.

The U.S. Navy keeps a continuous log of when radars are on, off, or broken. Figure 3 shows an actual usage history, over a two-month period, for a radar on one of the Navy's ships. Notice that at some points, on-off cycling is frequent. Also, some spells of data are missing. Data were collected on 45 radars of a particular model for a two-year period, yielding a total of 64 unit-years of complete data.

Although several features of the usage pattern may influence failures, this section focuses on a few simple covariates for exposition. For the Weibull model, two time-fixed covariates were defined (they remain constant over each on period) — percent time on and on-off cycles per day. Both were measured from the last repair until the start of the on period. For periods immediately following repair, both were set to zero. The log of the time-since-on was used as a time-dependent covariate, and time-since-repair was used as a time index. For the logit model, two additional covariates were used — length of the off period and time between the last repair and the start of the off period.

Table 1 presents some summary statistics of the data. Note that failures are much more likely while a machine is on than off. Also, on periods tend to be over three times as long as off periods. For a handful of observations, the on-off cycling rate covariates were extremely large (time-since-repair was small for these) and had a marked influence on the parameter estimates. These observations were deleted and are not included in table 1.

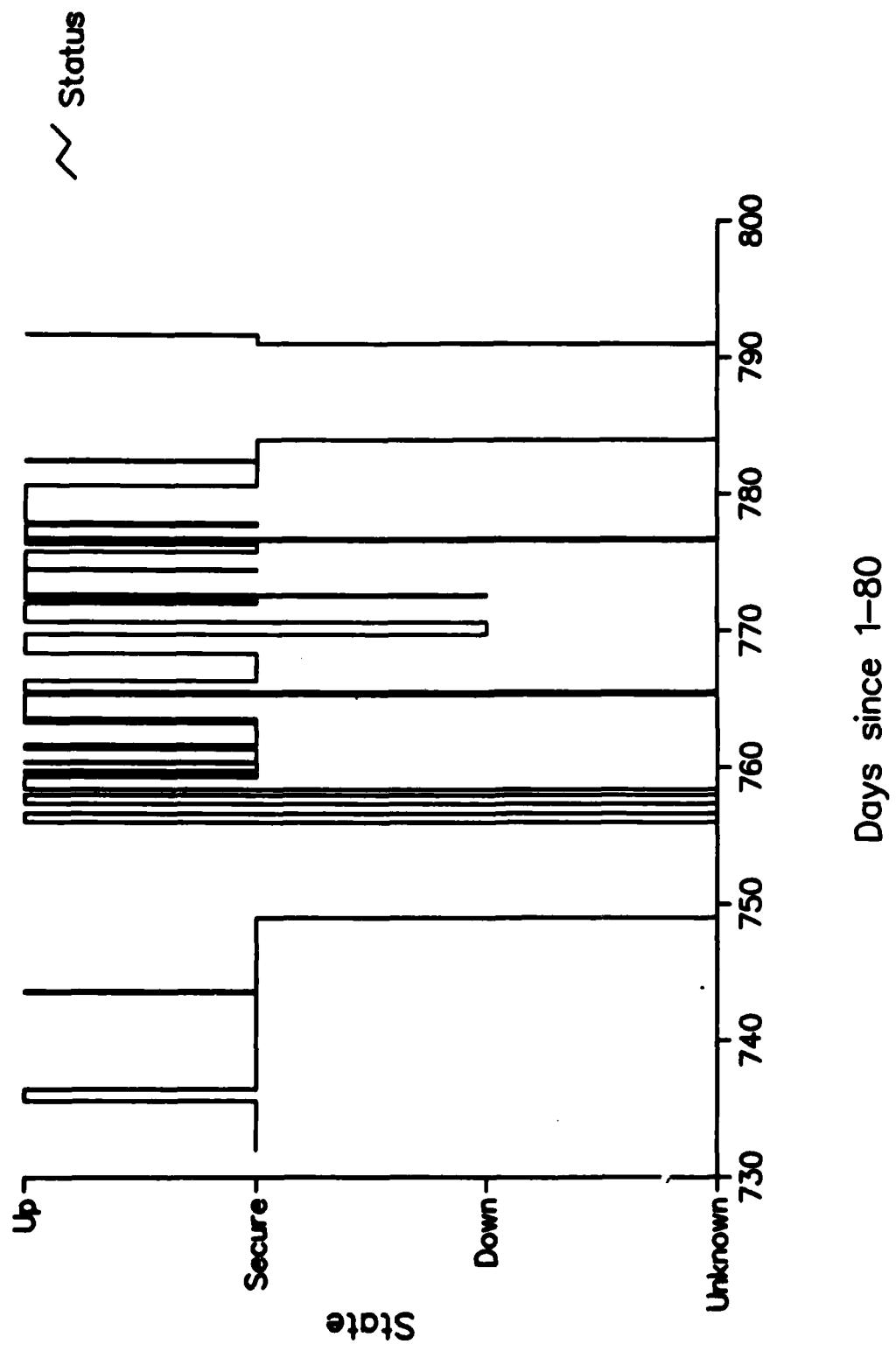


FIG. 3: ACTUAL USAGE HISTORY FOR ONE RADAR

TABLE 1
MEANS, STANDARD DEVIATIONS, AND COUNTS
OF SOME VARIABLES BY PERIOD

	Period	
	On	Off
Length of period in days	3.51 (5.50) ^a	1.00 (2.37)
Days since repair	13.80 (20.17)	18.24 (21.43)
On-off cycles per day since repair	.72 (1.14)	.79 (.82)
Percent time on since repair	.51 (.39)	.78 (.27)
Number of periods	2204	1695
Number of periods ending in failure	509	49

a. Numbers in parentheses are standard deviations.

Table 2 presents the maximum likelihood estimates and standard errors based on the two models. For the logit model, only the length of the off period has a statistically significant effect—the longer a machine is shut off, the less likely it is to work when an attempt is made to turn it on. For the Weibull model, all of the estimates are statistically significant. The signs of the regression estimates indicate that (1) machines that are used most of the time (since last repair) are less likely to fail, and (2) historical on-off cycling increases the chance of failure. The Weibull shape parameters indicate that risk decreases following repair and also following switching on. Thus, while a machine is turned on, on-off cycling has two harmful effects: an immediate higher risk as described by α_2 and a cumulative higher risk. Although when a machine is turned off there is no estimated cumulative effect of on-off cycling, each off period carries its own cost. Every time the machine is shut off, there is a small probability of failure.

TABLE 2
PARAMETER ESTIMATES

Effect	Model	
	Two-Weibull	Logit
α_1 (time since repair)	.878 (.04) ^a	-
α_2 (time since on)	.871 (.04)	-
Intercept	-2.259 (.07)	-3.883 (.58)
Time since repair	-	.012 (.01)
Length of off period	-	.067 (.03)
On-off cycles per day since repair	.114 (.04)	-.070 (.19)
Percent time on since repair	-.400 (.15)	.665 (.60)

a. Numbers in parentheses are the standard errors.

As an aid to interpreting the parameter estimates, figure 4 combines the continuous hazard (for on periods) and the probability of failure or discrete hazard (for off periods) for a short usage pattern. In figure 4, the radar is turned on and then shut off on alternate days (following repair) over a four-day period. The continuous hazard rate decreases sharply for the first day and decreases less steeply for the third day. The probability of failure is quite small for both off periods.

Model fit is assessed using the techniques discussed in the previous section. Figure 5 displays a plot of the deviance residuals against grouped cell number. Quintiles for the first two covariates (secure period and time since failure) were computed, and each observation was grouped into one of the cells defined by the 25 pairs of quintiles. Almost all of the r_k s are less than two³ in absolute value, suggesting the logit cannot be dismissed as inappropriate. Figure 6 displays a smoothed plot of the Kaplan Meier estimate of

³Recall that the r_k s have roughly a standard normal distribution if the model fits.

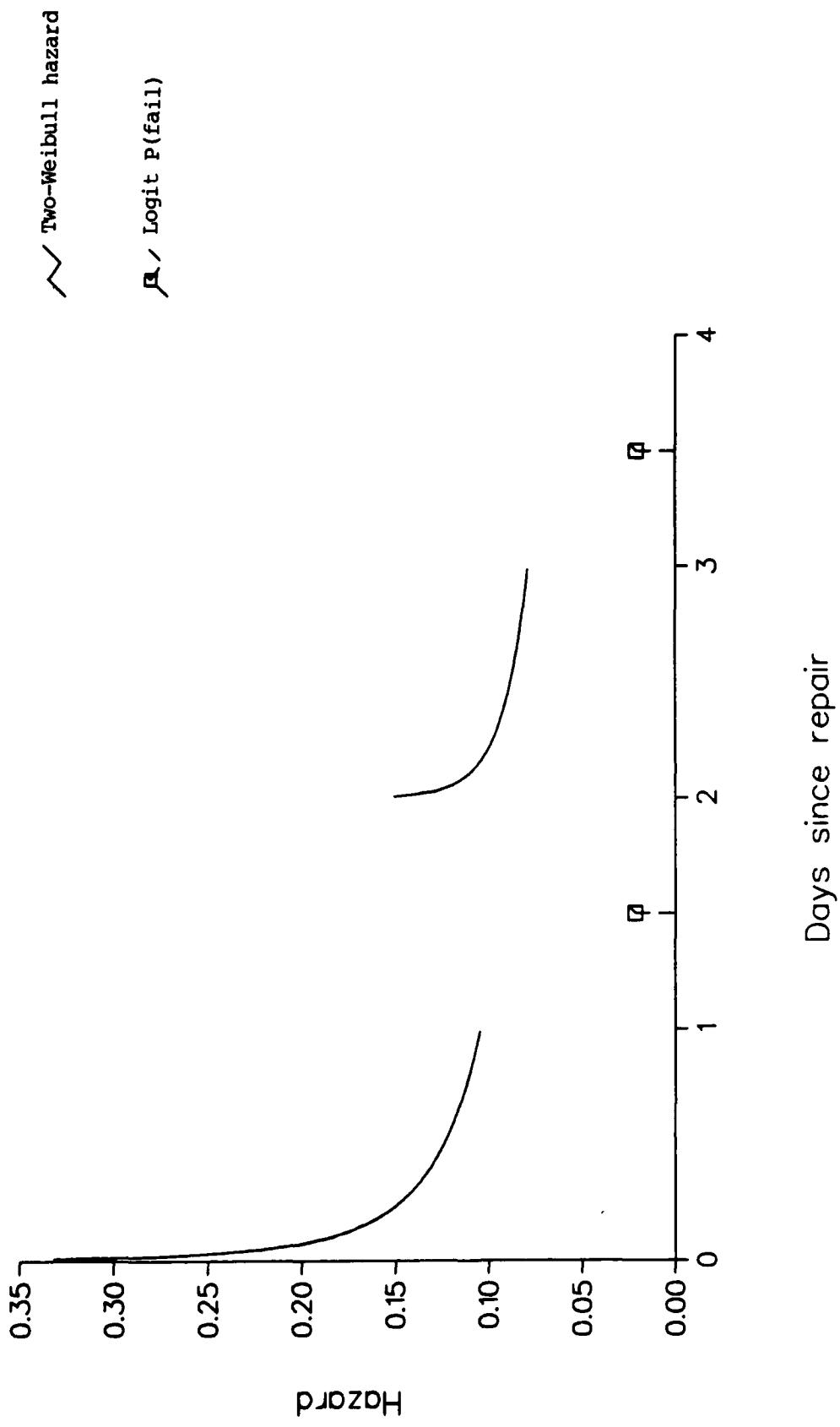


FIG. 4: ESTIMATED HAZARD FUNCTION FOR A DAILY-ON/DAILY-OFF PATTERN

the $\hat{F}(u_{i,j})$ s. The curve roughly follows that of a uniform(0,1) survival function, which indicates the two-Weibull model fits reasonably well.

Having assessed model fit, a measure of confidence concerning predictions from the model is warranted. Although caution against extrapolation to unusual usage patterns must be exercised, prediction can help decide on usage. If there are several possible usage patterns, the probability of failure under each pattern can be computed and the one with the lowest probability of failure chosen. As an illustration, suppose preventive maintenance (PM) over a two-week period is planned for the radar and that two programs are possible: (1) the radar is shut off for one hour each day for PM, or (2) the radar is shut off for two hours every other day for PM. For simplicity, assume that at all other times the radar is working and that the radar has just been repaired. In both cases, the radar is on for all but 14 hours; however, the on-off cycling rate for (1) is twice that of (2). With bidaily PM, the probability of no failure is .29; with daily PM, the probability of no failure is .19. The difference in probabilities suggests that bidaily PM would produce fewer failures, as long as the beneficial effect of daily versus bidaily PM was slight.

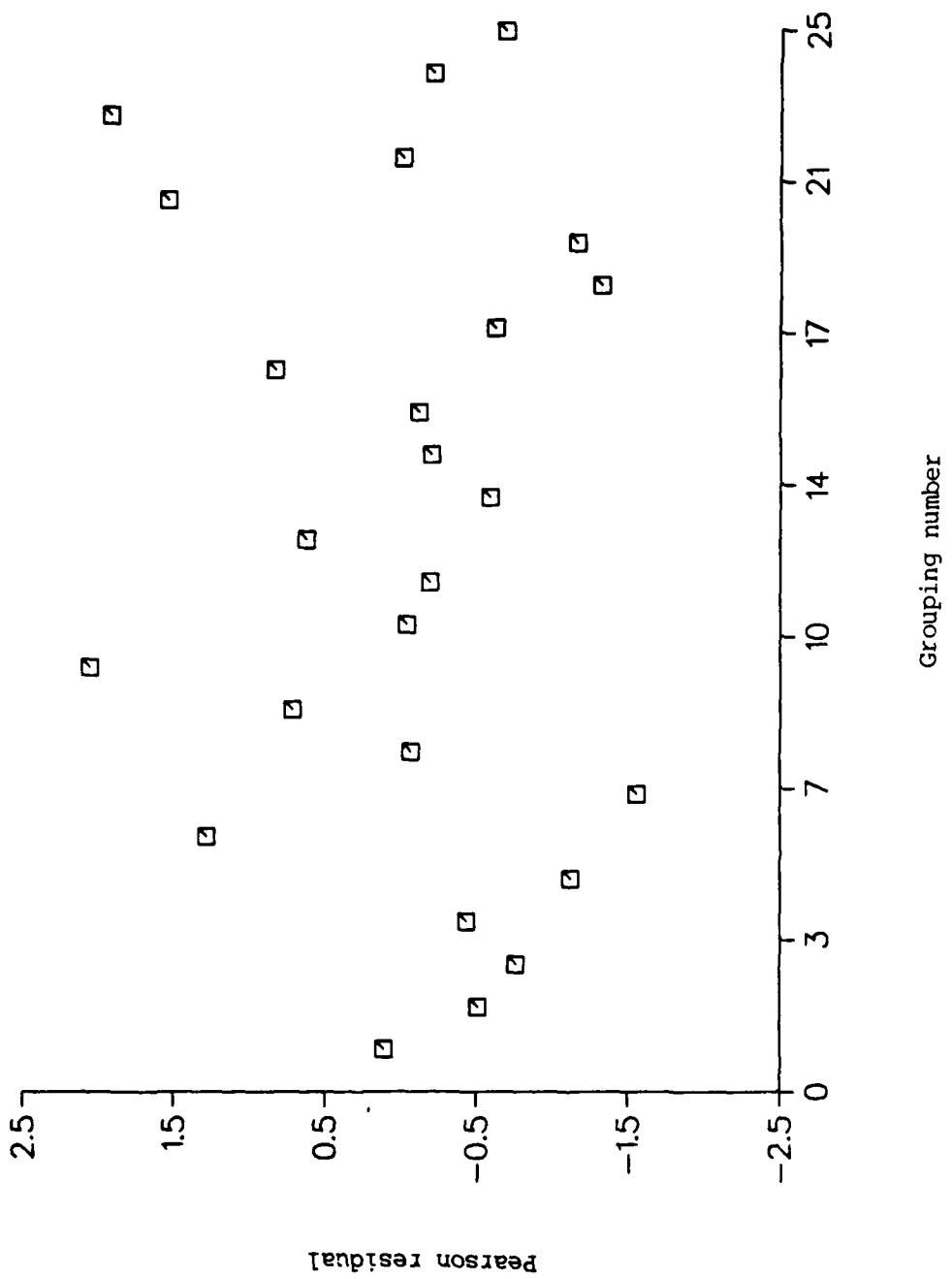


FIG. 5: PEARSON RESIDUAL BY GROUPING NUMBER

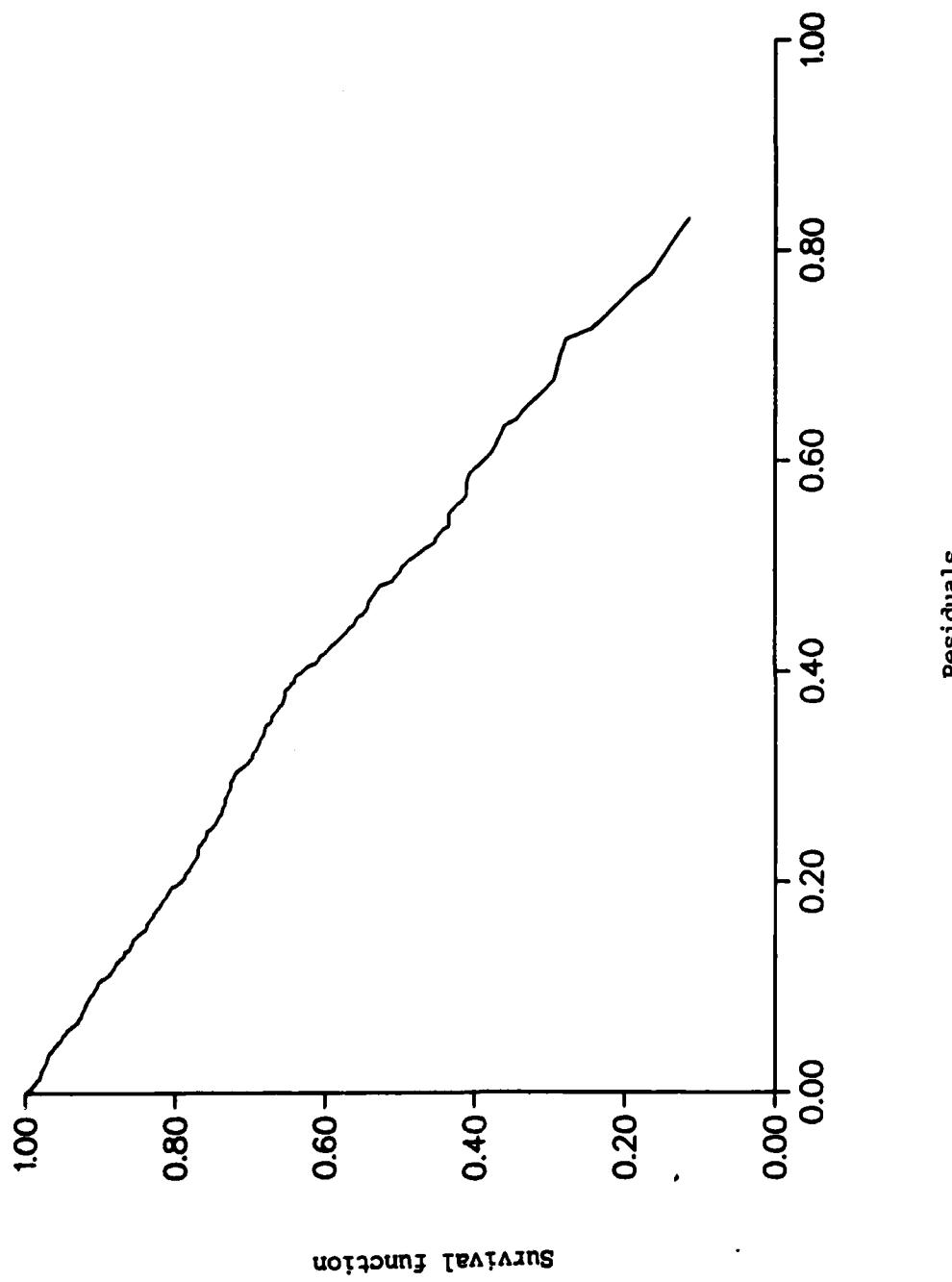


FIG. 6: ESTIMATED SURVIVAL FUNCTION OF THE RESIDUALS

DISCUSSION

This paper provides a parametric methodology for modeling failures of machines that are used intermittently. The hazard function of the time until failure is broken up into continuous and discrete components. These correspond, respectively, to the hazard during on and off periods. Other models for failures during these periods could be used. For example, a probit could be used during off periods or the baseline hazard could be left unspecified during on periods, yielding the usual Cox regression model [7]. In addition, a time index other than time since repair might have been chosen or a function other than *log* for the time-dependent covariate might be used. The point of this paper is to suggest a type of methodology for intermittently used machines where separate models are used for on and off periods.

The arguments leading to the two-Weibull model were to treat time-since-repair (*t*) as a time index and use a function of the time-since-on (*u*) as a time-dependent covariate. The model was shown to be equivalent to a Weibull with *u* as time index and a function of *t* a time-dependent covariate. Taking this equivalence one step further, perhaps it is not unreasonable to use *t* as a time-fixed covariate. This results in a Weibull regression on *u* with no time-dependent covariates and a much simpler maximization problem. Such an approach, of course, needs to be empirically justified.

Although the focus of this paper has been on intermittently used machines, the methodology could be applied to other situations where the time-until-failure is a combination of discrete and continuous components. Generalization to more than two states is straightforward: separate failure models could be specified for each state. Depending on how similar the different states are, some of the parameters for the different state models might be assumed to be equal.

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